

**MATHEMATICS - II  
(MATH 1201)**

**Time Allotted : 3 hrs**

**Full Marks : 70**

*Figures out of the right margin indicate full marks.*

*Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.*

*Candidates are required to give answer in their own words as far as practicable.*

**Group - A  
(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A fair coin is tossed thrice. The probability of getting at least two heads is  
(a)  $\frac{1}{2}$  (b)  $\frac{1}{4}$  (c)  $\frac{3}{8}$  (d)  $\frac{1}{8}$

(ii) The value of  $\beta\left(-\frac{3}{2}, \frac{7}{2}\right)$  is  
(a)  $-\frac{5\pi}{2}$  (b)  $\frac{5\pi}{2}$  (c)  $\frac{5\pi}{4}$  (d)  $\sqrt{\pi}$

(iii) If for a random variable  $X$ ,  $Var(X) = 1$ , then  $Var(-3X + 4)$  is  
(a) 4 (b) 13 (c) 9 (d) -3

(iv) One of the roots of  $x^2 + 5x - 3 = 0$  lies between  
(a) 0 and 1 (b) 1 and 2 (c) 2 and 3 (d) 3 and 4

(v) If  $X$  is normally distributed with zero mean and unit variance, then  $E(X^2)$  is  
(a) 0 (b) 1 (c) 2 (d) -1

(vi)  $\mathcal{L}\{t \sin 3t\}$  is  
(a)  $-\frac{6s}{(s^2 + 9)^2}$  (b)  $\frac{6s}{(s^2 + 9)^2}$  (c)  $\frac{6}{(s^2 + 9)^2}$  (d)  $\frac{3}{(s^2 + 9)}$

(vii) A binary tree has exactly  
(a) two vertices of degree two (b) one vertex of degree two  
(c) one vertex of degree one (d) two vertices of degree one.

(viii) Which of the following methods follows a predictor-corrector scheme?  
(a) Euler's method (b) R-K method of order 4  
(c) Modified Euler's method (d) Bisection method.

- (ix)  $\int_{-\infty}^{\infty} xe^{-x^2} dx =$   
(a) -1 (b) 0 (c) 1 (d) -2
- (x) To find a minimal spanning tree, the following algorithm is used:  
(a) Dijkstra (b) BFS (c) DFS (d) Prim.

**Group - B**

2. (a) There are two identical urns containing 4 white, 3 red balls and 3 white 7 red balls respectively. An urn is chosen at random and a ball is drawn from it. If the ball drawn is white, then what is the probability that it is from the first urn?
- (b) The mean weight of 500 male students at a certain college is 150 lbs and the standard deviation is 15 lbs. Assuming that the weights are normally distributed, find how many students weigh (i) between 120 and 155 lbs, (ii) more than 150 lbs.

**6 + 6 = 12**

3. (a) A random variable  $X$  has the following probability mass function

$X$	-2	-1	0	1	2	3
$P(X = x)$	0.1	$k$	0.2	$2k$	0.3	$3k$

- (i) Find the value of  $k$ .  
(ii) Obtain the distribution function  $F(x)$  of  $X$ .  
(iii) Find the mean and variance of  $X$ .

(b) A random variable  $X$  follows binomial distribution with mean 4 and standard deviation  $\sqrt{2}$ . Find the probability that the variable assumes a non zero value.

**7 + 5 = 12**

**Group - C**

4. (a) Find a real root of the equation  $x - \sin x = \frac{1}{2}$  by Newton-Raphson method, correct to four decimal places.
- (b) Solve the following system of equations by Gauss-Seidel method, correct to four decimal places:

$$3x + 4y + 15z = 54.8$$

$$x + 12y + z = 39.66$$

$$10x + y - 2z = 7.74$$

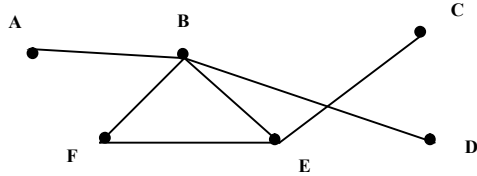
**5 + 7 = 12**

5. (a) Find a real root of the equation  $3x - \cos x - 1 = 0$ , by Regula-Falsi method, correct to three significant figures.
- (b) Using Euler's modified method, find an approximate value of  $y$  corresponding to  $x = 2.1$ , correct upto four decimal places given that  $\frac{dy}{dx} = 1 - \frac{y}{x}$  and  $h = 0.05$ .

6 + 6 = 12

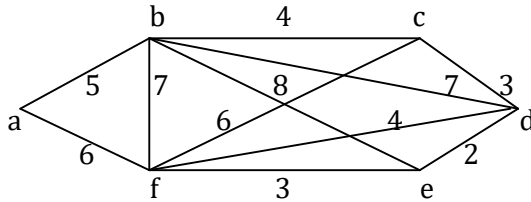
**Group - D**

6. (a) If a graph  $G$  and its complement  $G^c$  are both trees, find the order of  $G$ .
- (b) Does there exist a tree with 5 vertices where 2 vertices are of degree 3?
- (c) Find the incidence matrix of the following graph:



4 + 4 + 4 = 12

7. (a) Use Kruskal's algorithm to find a minimal spanning tree of the following graph.



- (b) Prove that the number of edges in a simple graph with  $n$  vertices cannot exceed  $\frac{n(n-1)}{2}$ .

7 + 5 = 12

**Group - E**

8. (a) Show that,  $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta = \frac{\pi}{\sqrt{2}}$

- (b) Evaluate  $\mathcal{L} \left\{ e^{-t} \left( \int_0^t t \cos t dt \right) \right\}$

6 + 6 = 12

9. (a) Evaluate  $\mathcal{L}^{-1} \left\{ \frac{1}{s^2(s+1)^2} \right\}$  by using the convolution theorem.

- (b) Solve the differential equation using the method of Laplace transform:  $y''(t) + 9y(t) = \cos 2t$ , where  $y(0) = 1, y(\pi/2) = -1$

5 + 7 = 12